

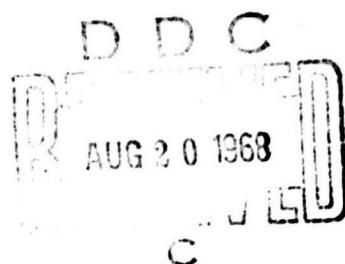
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A RESEARCH METHODOLOGY FOR STUDYING
COMPLEX SERVICE SYSTEMS

by

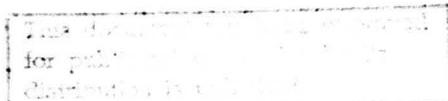
ROSSER T. NELSON

July, 1968



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Rosser T. Nelson¹

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This working paper is preliminary in nature, and subject to revision before publication in the open literature. It should not be quoted without prior consent of the author. Comments are cordially invited.

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PRELIMINARY REMARK

Because of the nature of this paper no SUMMARY Section is included for the "busy reader." To those readers who venture beyond the ABSTRACT, I feel obliged to recall² the words:

Believe nothing, no matter where you read it,
or who said it,
no matter if I have said it,
unless it agrees with your own reason and
your own common sense.

BUDDHA

ABSTRACT

This paper describes a research methodology for studying problems of analysis, design, and control in complex service systems. The proposed methodology is based upon the total systems point of view in the sense that the physical and decision-making aspects of the service system are considered and are related to environmental factors. The methodology was developed with two research areas in mind: (1) abstract studies of models of complex service systems and (2) field studies of

²I am not the first, e.g., see Bellman [2], page 194.

complex service systems. An example is included to illustrate the first type of application. Field studies may be expected to present far more of a challenge because of the identification, modeling, and data requirements which will arise in actual situations. The methodological plan presented here focuses attention on these requirements as an integral part of systems analysis.

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HISTORICAL OVERVIEW

The study of complex service systems has been an important research area in the production management field for some fifteen years. The earliest work centered upon abstractions of certain job-shop type production sequencing problems as static and deterministic combinatorial problems. Extensive research³ has been devoted to these combinatorial problems leading to extremely limited exact results [7] for small scale systems and a number of algorithms for feasibly generating approximate solutions for large scale systems [11,13]. Dissatisfaction with the static and deterministic nature of the combinatorial sequencing model led some researchers to the formulation of dynamic statistical queueing models for studying job-shop production systems as complex networks of queues. Some analytical results [6] have been obtained from queueing network models but computer simulation has evolved as the principle research method. The earliest simulation studies focused upon the evaluation of alternative queue disciplines or job sequencing priority rules for models with service centers as the only constraining resource [1,4,5]. More recent simulation models have included labor as a constraining resource and have been used to explore a wider range of design and control aspects of complex service systems [8,9,10,12]. A distinguishing characteristic of the research efforts in the field to date has been the use of relevant, but non-global, performance criteria

³Only representative historical references are given here. The reader interested in the history of job shop scheduling research and extensive reference material on the subject is referred to Chapter 11 of [3].

2.

such as time-in-system, job lateness, facilities utilization and in-process inventory. To the author's knowledge, this paper represents the first attempt to develop a research methodology which employs the physical service system model as an integral part of a total system, and which proposes to study the total system as it interacts with its environment.

I. INTRODUCTION

The evolution of operations research in the study of production systems has been marked by steady development in the speed and availability of computers and by increasing research and education in modeling, simulation, algorithms, and heuristics as techniques for analyzing and improving the performance of various aspects of complex production systems. At this stage of development, an important need exists for a procedural framework which blends the existing techniques into a research methodology for studying total systems. This paper is an attempt to respond to that need. In particular, the two purposes of this work may be stated as follows:

1. To propose a research methodology which considers the physical, environmental, and decision-making aspects of complex service systems as a total system, and, which employs existing operations research techniques as instruments to confront operational problems in a global context.
2. To present the methodology in generalized concepts and terminology in order to emphasize the potential applications to all systems which may be characterized as complex service systems.

The first stated purpose represents a natural extension of the production systems research described in the previous section. The second stated purpose is consistent with the recent trend among many leading educators in the production management area to focus on the broader concept of operations management as a functional area

exhibiting common problem types and methodology. This point of view is well described by Buffa [3] who emphasizes the importance in modern society of replacing the classification production system (so often taken as synonymous with manufacturing) by the classification productive system. "When we speak of productive systems we are thinking of more than simply physical production. If we adopt the economist's general definition, that is, 'production is the process by which goods and services are created', we are led to the broad view that productive systems include a tremendous range of activities in government, education, transportation and distribution, as well as manufacturing."

Two major areas of application are foreseen for the research methodology. The more obvious potential application is in field studies of specific service systems. A more inductive use of the methodology is also suggested; to formulate "characteristic" models with unspecified parameter values to describe classes of service systems and then predict system performance as a function of the unspecified parameters. Section 2 of the paper is devoted to a general description of the proposed methodology. Section 3 focuses on a particular example to which the methodology has been applied. The example is intended to serve two purposes; (1) to demonstrate the feasibility of the approach and its potential value in research applications of the second type described above, and, (2) to illustrate how the methodology brings to the fore the need for confronting the most difficult aspects of modeling the total system and the concomitant data requirements.

2. RESEARCH METHODOLOGY

In this section we wish to describe the proposed research methodology for studying complex service systems in a very general context. The reason for this generality is that we do not want to risk confusion between the methodology itself (which may be suitable for many applications) and the use of the methodology in a particular example. The reader who feels a need for clarification of the general concepts used here is referred to the production system example which follows.

Figure 1 serves as a schematic diagram to structure our view of the physical service system, intra-system decision making, and the environment. A system demand process imposes a demand for goods and (or) services upon the physical system. The physical system is characterized by design parameters and control parameters which describe the relevant physical attributes and operational procedures of the physical system. The output of the physical system is measured by system performance statistics. These performance statistics are observable (at least in part) directly by the environment. In addition, they serve as one input for the evaluation of the system criterion function and, in this way, contribute to the system decision process. The system decision process uses the system performance evaluation to make two types of decisions; decisions affecting the physical system through the system design and control variables and decisions tied to the environment through the system-environment decision variables. The environmental response process assesses the system-environment decisions and the system performance statistics and adjusts the system demand process accordingly. Thus the system

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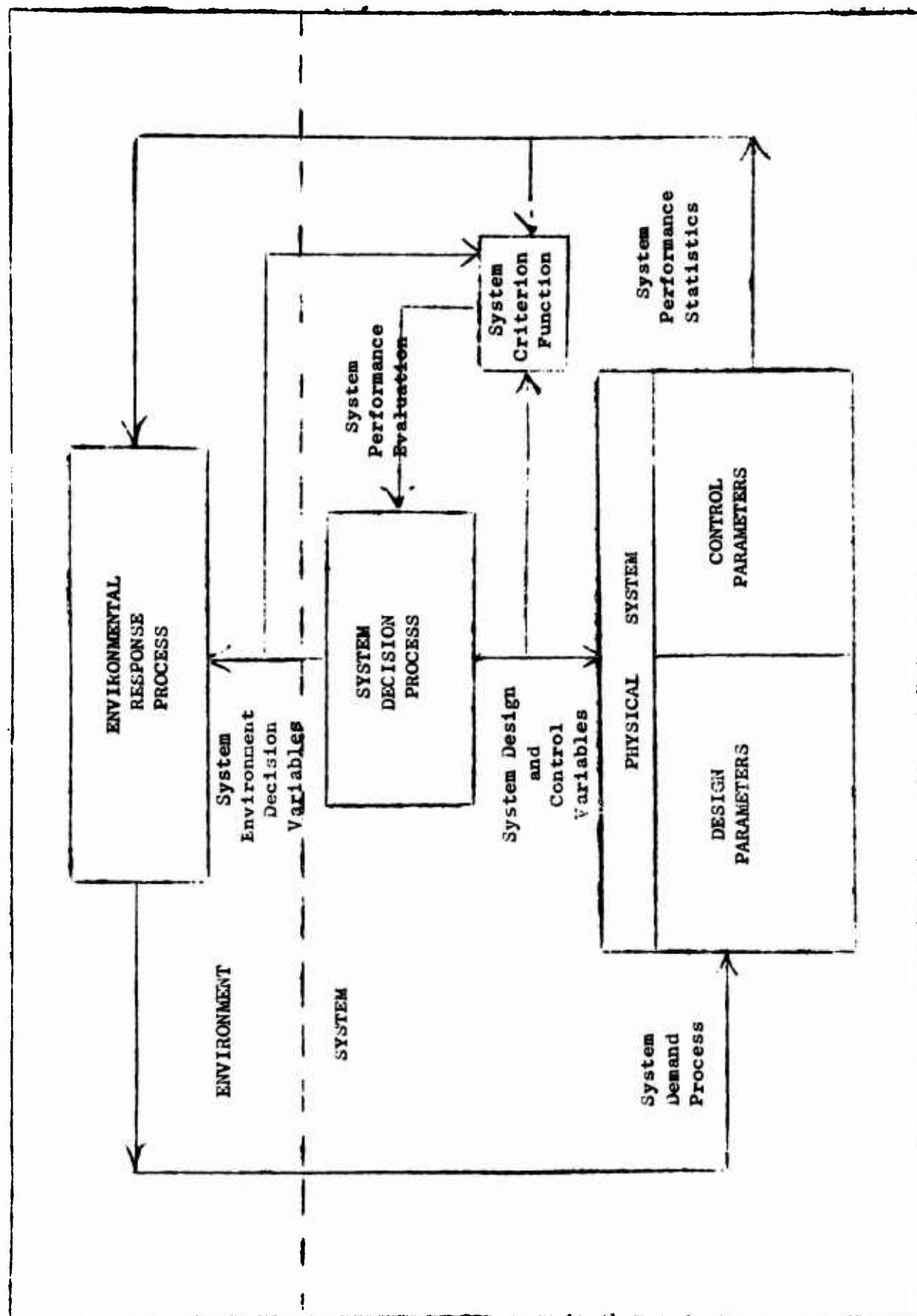


Figure 1. Generalized System Diagram

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and the environment are related by a dynamic, closed loop interaction.

For the reason given above, the outline and description of the proposed research methodology is given below in the general terminology of Figure 1. A translation of the terminology for a specific example is provided in Section 3 where the same step identification symbols are employed.

METHODOLOGICAL OUTLINE

A. Identification and Modelling

- A1. Modelling of the physical system**
- A2. Characterization of the system decision process**
- A3. Modelling of the system criterion function**
- A4. Modelling of the environmental response process**

B. Experimentation

- B1. Input-output analysis**
- B2. Equilibrium analysis**
- B3. System performance evaluation**
- B4. Variation of system-environment decision variables**
- B5. Variation of system control variables**
- B6. Variation of system design variables**

C. Implementation

A. Identification and Modelling

The first or pre-experimental phase of the methodology consists of modelling each of the four input-output boxes in Figure 1:

- A1. Modelling the physical system. This step requires the development of a simulation⁴ model of the physical service system and the identification and classification of the design and control parameters which may be used as decision variables in ensuing experimentation.
- A2. Characterization of the system decision process. The system decision process represents the regulatory action operating within the system. The decision process may be characterized by a formal model or algorithm which maps system performance evaluation into sets of decisions or, in the absence of such a structured decision-making procedure, it may be characterized by merely identifying the available decision variables--both those related to the physical system and those acting upon the environment.⁵ In the former case, the revision of decisions with time is accomplished according to a specified algorithm which takes the place of steps B4 through B6 below. In the latter case, alternative decisions are employed as free variables in the experimentation as described in Steps B4 through B6.

⁴ Throughout the paper we shall assume that the service system under study is sufficiently complex to rule out consideration of a model amenable to analytical solution.

⁵ Step A2 is isolated here for orderly presentation. In practice, of course, it is an integral part of steps A1 and A4.

A3. Modelling of the system criterion function. This step requires the construction of a model for the evaluation of system performance.

A4. Modelling of the environmental response process. This step requires the formulation of a model which describes how system performance statistics and system-environment decision variables combine to determine the system demand process. Data collection and stationarity problems are most likely to arise in this area because the response process is external to the system and, in general, is subject to many influences from other sources than the system under study. Time monitoring of the environmental response process may be necessary to assure that the response model represents current conditions.

B. Experimentation

B1. Input-output analysis. This step involves the simulation of the physical system using a fixed set of design and control parameters. The objective of the simulation is to obtain the relationships between the system demand process (input) and the system performance statistics (output).

B2. Equilibrium analysis. For a fixed set of system-environment decision /parameters, the model of the environmental response process (Step A4) describes the system demand process as a function of the system performance statistics. The input-output analysis (Step B1) experimentally related the system

performance statistics to the system demand process.

Combining these two sets of relationships makes it possible to determine whether the fixed physical system and the fixed set of system-environment decisions are a feasible combination and, if feasible, to determine the operating conditions which represent equilibrium between the system and its environment.

- B3. System performance evaluation. This step consists of the development of the system criterion function and the subsequent evaluation of system performance for the equilibrium operating conditions obtained in Step B2. This evaluation applies only to the fixed set of design and control parameters and the fixed set of system-environment decision parameters employed in Steps B1 and B2.
- B4. Variation of system-environment decision variables. This phase of the experimentation consists of the repetition of Steps B1 through B3 in order to evaluate alternate sets of system-environment decision parameters in conjunction with the fixed set of system design and control parameters.
- B5. Variation of system control variables. This phase of the experimentation consists of the repetition of Steps B1 through B4 in order to evaluate alternative system control procedures.
- B6. Variation of system design variables. This phase of the experimentation consists of the repetition of Steps B1 through B5 in order to evaluate alternative system designs.

C. Implementation

We assume that the operating system and its environment are sufficiently complex and variable with time that the possibility of optimization is not to be taken seriously. Instead, the proposed methodology is based on the objectives of currency and systematic improvement. Implementation of the methodology will require interaction between the operating system and the system model in combination with continuing application of the methodology. Figure 2 is intended to illustrate the process which includes continuing observation of the operating system (and its environment) and modification of the system model as well as continuing experimentation or application of the decision algorithm.

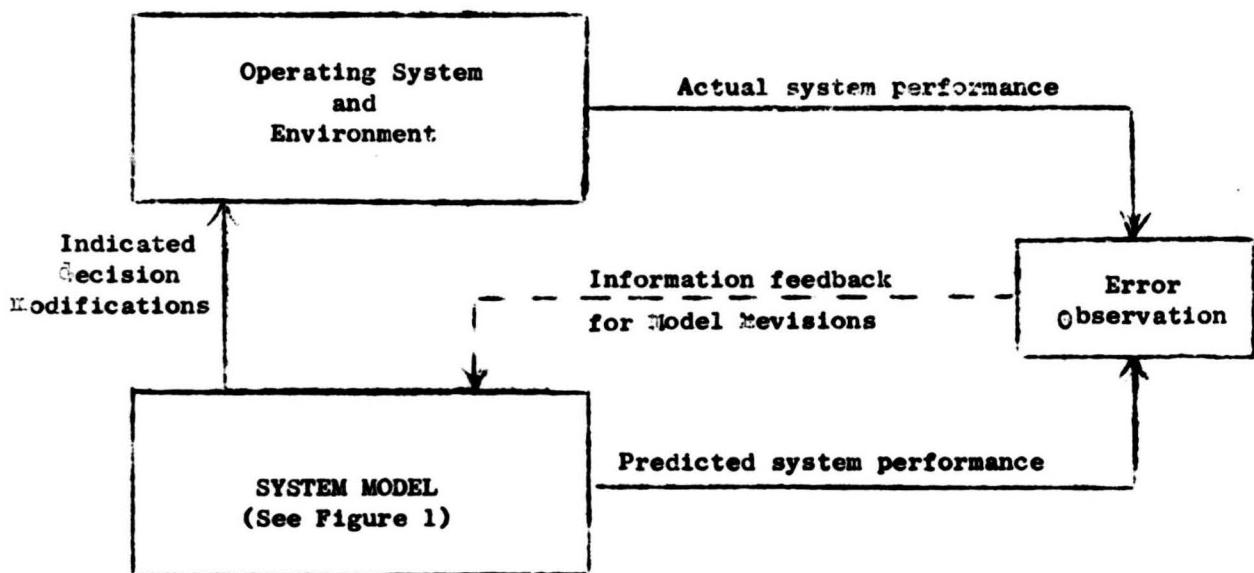


Figure 2. Implementation Diagram

Experimentation with the system model leads to indicated decision modifications as inputs to the operating system. The frequency with which these occur will depend upon the time and cost required to experiment with the system model. Another relevant factor is the efficiency of the experimental design or algorithm which represents the system decision process within the system model. The adoption of indicated decision modifications in the operating system will depend upon the costs associated with the indicated changes as compared to the predicted improvement in system performance. For the set of operating decisions in use at any time, the actual and predicted system performance are compared. The purpose of this comparison is to detect differences which indicate the failure of the system model to adequately represent the current nature of the operating system. The information fed back from this observation process is the basis for investigating the necessary areas for parameter changes and (or) more basic revisions in the system model.

3. AN EXAMPLE

To illustrate the use of the methodology proposed in Section 2, we shall consider a simple production-inspection system producing a single product to inventory. This example was selected as a starting point for two reasons. First, it serves as an example for the purposes of this paper. Second, as we shall see in the discussion which follows, the single product to inventory assumptions make the development of a representative cost model relatively easy and lead to a constant arrival interval, constant service time model which is a natural starting point for continuing research employing the methodology.

PROCEDURAL OUTLINE

We shall first describe the system and experimentation in detail by following the research methodology outline presented in Section 2. A presentation of illustrative results follows the procedural outline.

A. Identification and Modeling

A1. Modelling the physical system. A generalized simulation model of labor and machine limited production systems was used to represent the physical system. The general model is described in detail in [8]. The particular version of the model used for the physical system in this example is represented schematically in Figure 3.

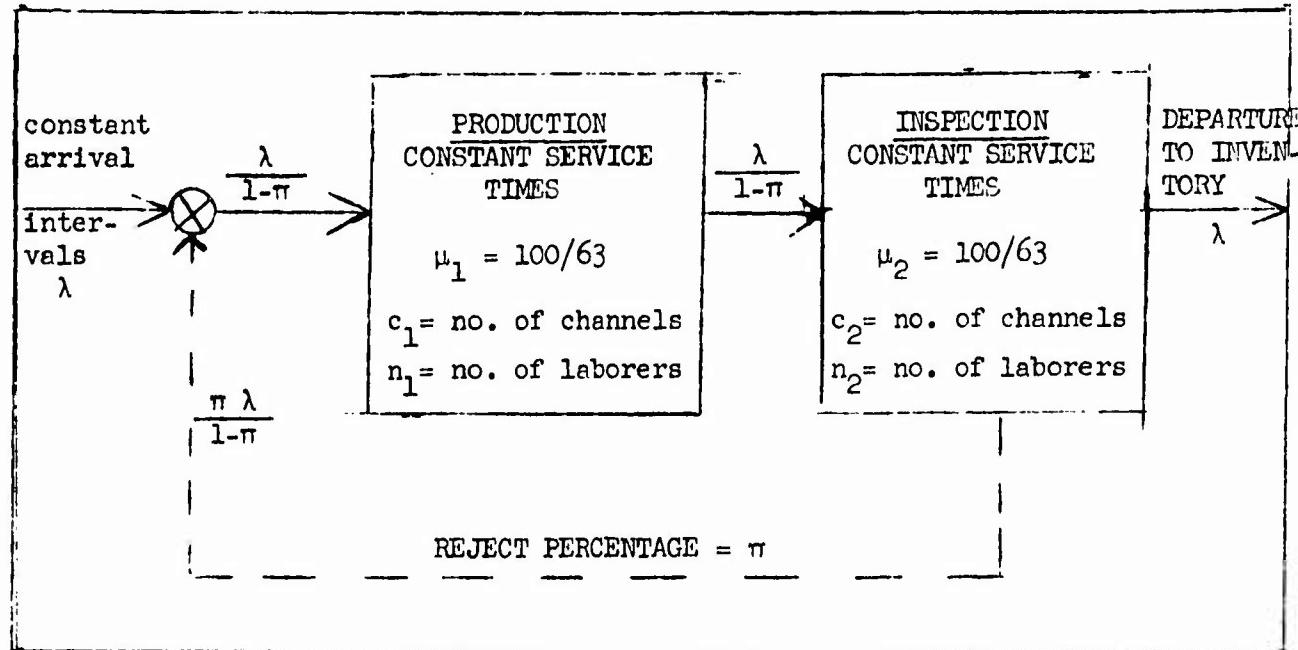


FIGURE 3. PHYSICAL SYSTEM

Since production is to inventory, the beginning of new units of product is taken to be a constant arrival interval process with a mean arrival rate λ . The single product requires a fixed service time for production and a fixed service time for inspection. The inspection process is assumed to be a Bernoulli process with reject probability π . The rejection of a unit of product results in the generation of a re-order for a new unit of product at the production stage. The reject percentage (control) and the number of service channels and laborers in production and in inspection (design) are the only experimental variables in the example. Many other design and control variables are available in the simulation model. These are classified in Table 1 where the fixed parameters and variables are identified for the four experiments used in the example. A discussion of the rationale for the choice of experiments will be deferred until Part B of this section where the experimentation is described.

CLASSIFICATION OF DESIGN AND CONTROL VARIABLES ⁶	EXPERIMENT NUMBER			
	1a	1b	2a	2b
VARIABLES RELATIVELY EASILY SUBJECT TO SHORT-RANGE CONTROL				
q_1, q_2 - queue disciplines in production and inspection	Fixed: First-in-system, first-served.			
λ - machine center selection procedure for labor assignment	Fixed: Irrelevant for the labor efficiency matrix employed.			
VARIABLES REPRESENTING SHORT TO MEDIUM-RANGE CONTROL				
λ - mean arrival rate of system demand process	1.36	1.21	2.72	2.42
μ_1, μ_2 - mean service rates of production and inspection	Fixed: $\mu_1 = \mu_2 = 1.58$			
π - reject percentage from inspection	$\pi = 0$ Studied by queueing theory $\pi = .05$ in experiments			
E - labor efficiency matrix	Fixed: One completely efficient laborer for each service channel			
d_1, d_2 - degree of centralized control over labor assignment at production and inspection	Fixed: $d_1 = d_2 = 1$ (Fully centralized control)			
BASIC DESIGN VARIABLES				
c_1, c_2 - number of service channels at production and inspection	$c_1 = 1$ $c_2 = 1$	$c_1 = 1$ $c_2 = 1$	$c_1 = 2$ $c_2 = 2$	$c_1 = 2$ $c_2 = 2$
n - size of labor force	2	2	4	4
RELATIVELY UNCONTROLLABLE CHARACTERISTICS REFLECTING THE NATURE OF THE PRODUCTION PROCESS				
a(.) - arrival process density function	Fixed: Constant arrival intervals			
$s_1(.), s_2(.)$ - service time density functions of production and inspection	Fixed: Constant service times			
P - customer routing transition matrix	Fixed: See Figure 3			

TABLE 1. CLASSIFICATION OF DESIGN AND CONTROL VARIABLES IN THE SIMULATION MODEL OF THE PHYSICAL SYSTEM

⁶A detailed explanation of the design and control variables and their roles in the labor and machine limited simulation model is given in [8].

A2. Characterization of the system decision process. For the purposes of this example the available decision variables may be classified as follows:

Decision Variables Related To The Physical System

π - probability of rejection of each item produced

c_1, c_2 - number of service channels at production and inspection, respectively

n_1, n_2 - number of laborers assigned to production and inspection, respectively

Decision Variables Related To The Environment

k_1 - selling price per unit produced } market decision variables
 c_{12} - promotional expenditure rate

A3. Modelling of the system criterion function. For the production-inspection example, profit (see (14) below) was taken as the system criterion function. A detailed cost model was constructed to reflect the production of a single item to inventory. In addition to income from sales, the model includes the cost components (1) through (13) below:

(1) Income from sales (I)

$$I = k_1 \lambda \quad \left\{ \begin{array}{l} I: \text{income from sales - dollars per unit time} \\ \lambda: \text{mean production rate - items per unit time} \\ k_1: \text{selling price - dollars per item} \end{array} \right.$$

(2) Raw material costs (C_1)

$$C_1 = \frac{k_2 \lambda}{1-\pi} \quad \left\{ \begin{array}{l} C_1: \text{raw material costs - dollars per unit time} \\ \lambda: \text{mean production rate - items per unit time} \\ \pi: \text{reject percentage - dimensionless} \\ k_2: \text{unit raw material cost - dollars per item} \end{array} \right.$$

(3) Equipment costs (C_2)
$$C_2 = k_3 c_1 + k_4 c_2$$

c_2 : equipment costs - dollars per unit time
 c_1 : number of service channels for production - channels
 c_2 : number of service channels for inspection - channels
 k_3 : cost per channel for production - dollars per channel per unit time
 k_4 : cost per channel for inspection - dollars per channel per unit time

(4) Labor costs (C_3)
$$C_3 = k_5 n_1 + k_6 n_2$$

c_3 : labor costs - dollars per unit time
 n_1 : number of workers assigned to production - men
 n_2 : number of workers assigned to inspection - men
 k_5 : labor cost of production workers - dollars per man per unit time
 k_6 : labor cost of inspection workers - dollars per man per unit time

(5) Raw material inventory costs (C_4)
$$C_4 = \frac{k_7 \lambda t_R}{2(1-\pi)} + k_7 b_R$$

c_4 : raw material inventory costs - dollars per unit time
 λ : mean production rate - items per unit time
 t_R : raw material lead time - time units
 π : reject percentage - dimensionless
 k_7 : raw material storage cost rate - dollars per item per unit time
 b_R : raw material buffer stock level - items

(6) In-process inventory costs (C_5)

$$C_5 = k_8 \lambda \bar{f}$$

$\left. \begin{array}{l} C_5: \text{average in-process inventory costs - dollars per unit time} \\ \lambda: \text{mean production rate - items per unit time} \\ \bar{f}: \text{mean time in production - inspection system - time units} \\ k_8: \text{in-process inventory carrying cost rate - dollars per item per unit time} \end{array} \right\}$

(7) Finished goods inventory costs (C_6)

$$C_6 = k_9 b_F$$

$\left. \begin{array}{l} C_6: \text{finished goods inventory costs - dollars per unit time} \\ b_F: \text{finished goods buffer stock level - items} \\ k_9: \text{finished goods carrying cost rate - dollars per item per unit time} \end{array} \right\}$

CONTROL COSTS. The control costs C_7 through C_{10} are constants when working with a fixed set of control parameters. When the control parameters of the physical system are varied in experimentation, these constants must be appropriately modified to compare alternative control procedures. Explicit relationships between the control costs and control parameters would be an inherent part of the cost model in actual applications. For the purposes of this example, we shall merely indicate implicit functional dependencies.

(8) Labor training costs (C_7)

$$C_7 = k_{10} = \text{function } (E)$$

$\left. \begin{array}{l} k_{10}: \text{labor training costs - dollars per unit time} \\ E: \text{labor efficiency matrix} \end{array} \right\}$

(9) Labor assignment procedure costs (c_8)

$$c_8 = k_{11} = \text{function}(\ell, q, d)$$

$\left\{ \begin{array}{l} k_{11}: \text{labor assignment procedure costs - dollars per unit time} \\ \ell, q, d: \text{labor assignment procedure descriptors} \\ \quad (\text{See Table 1}) \end{array} \right.$

(10) Process control (e.g., maintenance, quality control) costs (c_9)

$$c_9 = k_{12} = \text{function}(\mu_1, \mu_2, \pi)$$

$\left\{ \begin{array}{l} k_{12}: \text{process control costs - dollars per unit time} \\ \mu_1, \mu_2: \text{mean service rates of production and inspection} \\ \pi: \text{reject percentage} \end{array} \right.$

(11) Centralized control costs (c_{10})

$$c_{10} = k_{13} = \text{function}(d_1, d_2)$$

$\left\{ \begin{array}{l} k_{13}: \text{centralized control costs - dollars per unit time} \\ d_1, d_2: \text{degree of central control parameters} \\ \quad (\text{See Table 1}) \end{array} \right.$

(12) Customer goodwill costs (c_{11})

$c_{11} = 0$ a cost reflected indirectly by the dependence of the demand rate upon the system performance statistics

(13) Promotional costs (c_{12})

$$c_{12} = \text{function}(\lambda, k_1, \bar{f})$$

$\left\{ \begin{array}{l} c_{12}: \text{promotional costs - dollars per unit time} \\ \lambda: \text{mean production rate - items per unit time} \\ k_1: \text{selling price - dollars per item} \\ \bar{f}: \text{mean time in production - inspection system-time units} \end{array} \right.$

NOTE: The above equation arises from the demand model of the form
which is discussed in the next section.

(14) Profit equation (P: profit rate - dollars per unit time)

$$P = k_1 \lambda - \frac{k_2 \lambda}{1-\pi} - k_3 c_1 - k_4 c_2 - k_5 n_1 - k_6 n_2 - k_7 \left[\frac{\lambda t_R}{b_R + 2(1-\pi)} \right] - k_8 \lambda \bar{f} \\ - k_9 b_F - k_{10} - k_{11} - k_{12} - k_{13} - c_{12} (\lambda, k_1, \bar{f}).$$

A4. Modelling of the environmental response process. Because the environmental response process is based on factors external to the system, this facet of the methodology may often prove challenging. The systematic inquiry and data collection required may, in itself, lead to better understanding of how the system interacts with the environment. For our example, the production of a single product to inventory lends itself to a relatively simple model of the environmental response process; in this case, the market demand process for the single product. In particular, the maintenance of an average buffer stock level b_F in finished goods inventory [see component (7) of the cost model] enables the production of individual items to be initiated at uniform time intervals with the buffer stock absorbing the statistical fluctuations in demand. Thus, the system demand process may be viewed as a constant arrival interval process described by the single parameter λ (the arrival rate). The finished goods average buffer stock level also has a direct effect on the system performance statistics which are relevant to the customer. There are three typical cases:

- (1) $b_F=0$ In this case each customer must wait for his individual order to be produced. Each customer is therefore affected by both the mean time in the production system and the variability of the time-in-system among different items.

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(2) b_F =large In this case each customer is satisfied immediately upon order from the buffer stock. Customers are not affected at all by time-in-system statistics.

(3) b_F =intermediate In this case each customer waits for the mean time-in-system for his order to be produced but the buffer stock is used so that individual customers are not subject to the variability of time-in-system.

For the purposes of the example, we shall confine attention to cases (2) and (3), i.e., we shall assume b_F is sufficient to absorb the variable time-in-system for individual items.

Based on the above considerations, the model of the market demand process to be used is given by:

product demand rate=function (market decision variables, mean time-in-system)

$$\lambda = \text{function}(k_1, C_{12}, \bar{f}) \left\{ \begin{array}{l} k_1 : \text{selling price - dollars per item} \\ C_{12} : \text{promotional costs - dollars per unit time} \\ \bar{f} : \text{mean time-in-system - time units} \end{array} \right.$$

Having now discussed each aspect of the identification and modeling for the example, we may construct the complete system diagram for the production example which corresponds to the generalized system diagram of Figure 1.

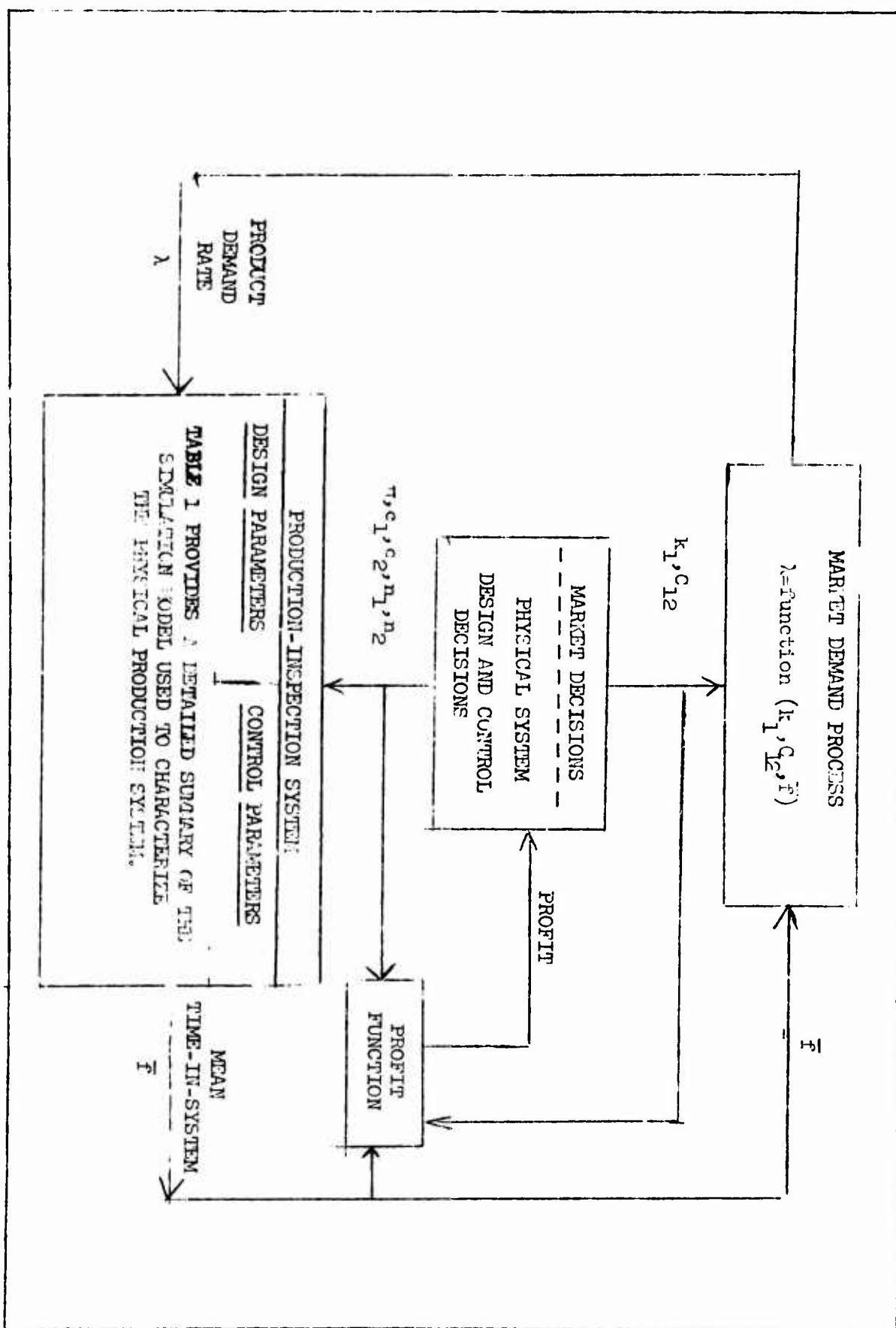


FIGURE 4. SYSTEM DIAGRAM - PRODUCTION EXAMPLE

B. EXPERIMENTATION⁷

B1. Input-output analysis. The fixed set of design and control variables used for Experiment 1 were $c_1=c_2=1$, $n_1=n_2=1$, $\pi=.05$. These correspond to a single service channel with a single laborer for production followed by a single service channel with a single laborer for inspection, and a reject rate of 5%. The system performance function, i.e., the relationship between the demand rate λ and the mean time-in-system \bar{f} was studied by simulation of the production system model. Values of λ giving average system utilizations of .9 and .8 were employed for experimental runs la and lb, respectively.

B2. Equilibrium Analysis. For the purposes of the example the market demand function was assumed to be of the form:

$$\lambda(k_1, c_{12}, \bar{f}) = 1 + \frac{2\sqrt{c_{12}} \exp(-k_1)}{\bar{f}}$$

Other factors fixed, the mean demand rate was assumed proportional to the square root of the promotion expense rate, exponentially decreasing with increasing selling price, and inversely proportional to mean time in system. For a fixed set of market decisions k_1 and c_{12} the equilibrium operating conditions λ and \bar{f} were obtained from the intersection of the system performance function from Step B1 (which gives \bar{f} as a function of λ) and the market demand function (which gives λ as a function of \bar{f}).

⁷Computations were performed on the IBM 7094 computer of the Campus Computing Center, UCLA.

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B3. System performance evaluation. The cost model developed in Step A2 was applied to obtain the profit for the system of Experiment 1 under the equilibrium operating conditions for λ and \bar{f} from Step B2. The result is an equation for profit in terms of the cost parameters k_2 through k_{13} and the inventory parameters b_R, t_R , and b_F , for the fixed market decisions k_1 and c_{12} .

B4. Variation of system-environment decision variables. Steps B1 through B3 were repeated in order to evaluate alternate sets of market decisions k_1 and c_{12} for the fixed physical system of Experiment 1. For each set of decisions considered, the equilibrium values of λ and \bar{f} corresponding to those decisions were used to obtain the profit expression. The incremental profit was obtained as a linear function of certain of the system cost parameters. This resulted in a linear inequality in terms of the system cost parameters that serves to determine which of the alternative sets of market decisions is superior.

B5. Variation of system control variables. Steps B1 through B4 were repeated after changing the reject percentage π from .05 to 0 in order to ascertain the value of perfect control of production quality⁸. The latter case being a simple ordinary constant arrival, constant service, series queueing system, it was not necessary to simulate the system to obtain the

⁸It is assumed that an inspection operation is still required when $\pi=0$ to insure product quality. Obviously, $\pi=0$ is a degenerate case which is used in the example merely to reduce the necessary computer simulation.

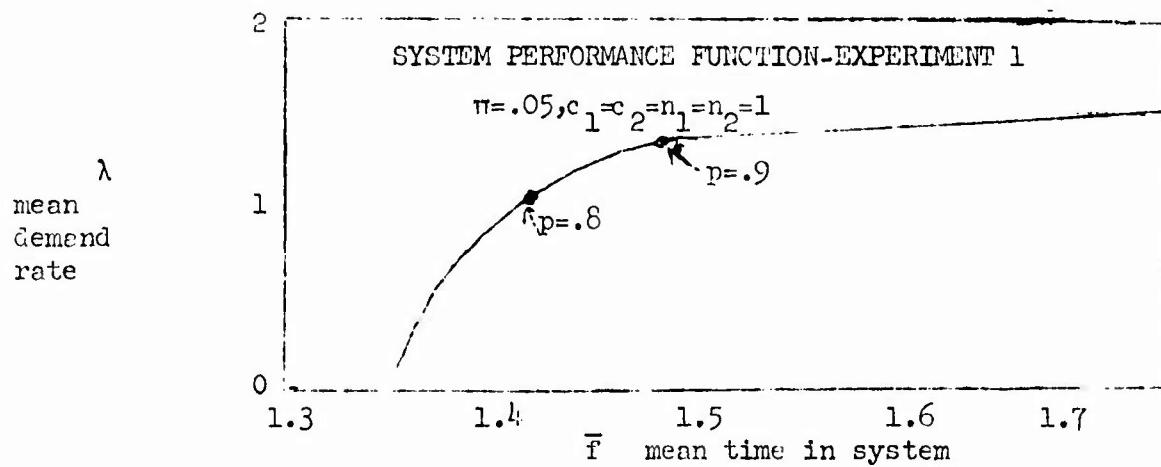
system performance function. The comparison of the two systems with different values of the control parameter π is based on comparison of the profit expression for $\pi=.05$ obtained experimentally and the profit expression for $\pi=0$ obtained by a purely analytical development. Again, a linear inequality in certain of the system cost parameters determines which of the two systems is best.

B6. Variation of system design variables. Experiment 2 consisted of a complete replication of Steps B1 through B5 with basic design changes in the physical system. In particular, an additional service channel and laborer were added in both production and inspection. Values of λ giving average system utilizations of .9 and .8 were again employed for experimental runs 2a and 2b, respectively. As before, the resulting profit expressions are used to obtain inequalities in the system cost parameters which provide the basis for evaluating the alternate system designs.

ILLUSTRATIVE RESULTS

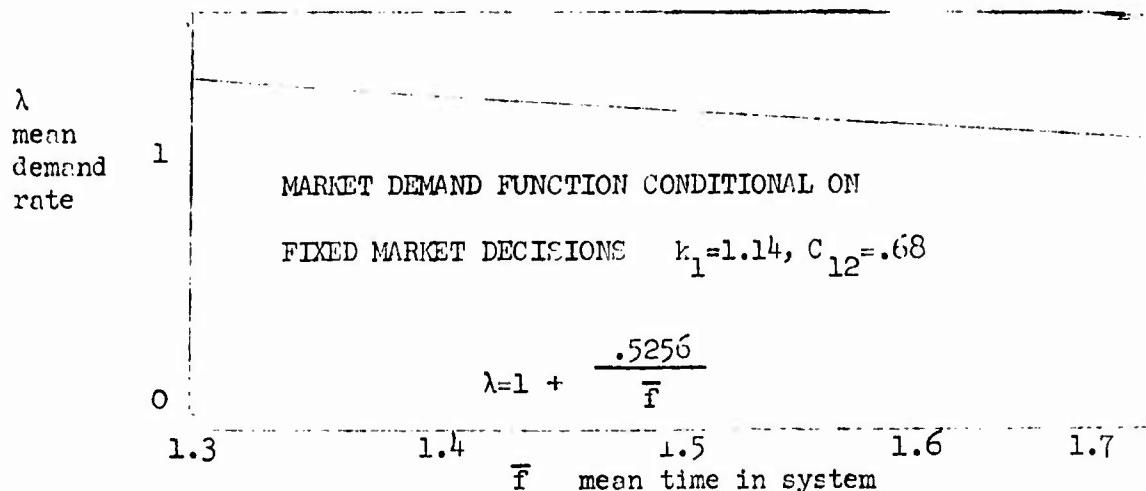
Step B1 - Input-Output Analysis.

The system performance function obtained from the simulations in Experiment 1 is given below:



Step B2 - Equilibrium Analysis.

The initial set of market decisions was taken to be $k_1=1.14$ and $c_{12}=.68$. The resulting market demand function conditional on the fixed market decisions is given below:



The equilibrium operating conditions for the fixed market decisions were found from the intersection of the system performance function and the market demand function. The equilibrium values are $\lambda=1.36$ and $\bar{f}=1.46$.

There are, of course, many sets of market decisions that would lead to this same set of equilibrium operating conditions. The set selected was that set which maximizes the profit expression derived from the cost model. The best set of market decisions for a set of operating conditions λ and \bar{f} (with the assumed market demand function) are given by:

$$k_1 = \frac{\ln \left[\frac{4\lambda}{2(\lambda-1)^2 \bar{f}^2} \right]}{2}, \quad c_{12} = \frac{\lambda}{2}$$

These relationships were used throughout the example to insure use of the best set of market decisions corresponding to different operating conditions. Hence, when the cost model is taken into account, the evaluation of alternative sets of market decisions becomes partly analytical and partly experimental.

Step B3 - System Performance Evaluation.

The cost model was applied to obtain the profit expression in terms of the system cost parameters k_2 through k_{13} and the system inventory parameters b_R, t_R , and b_F .

$$P = .875 - 1.432 k_2 - k_3 - k_4 - k_5 - k_6 - k_7 [b_R + .716t_R] - 1.986 k_8 \\ - k_9 b_F - k_{10} - k_{11} - k_{12} (\pi = .05) - k_{13}.$$

Step B4 - Evaluation of Alternative Market Decisions.

In order to evaluate an alternative set of market decisions, the formulas described in Step B2 were used to derive the best set of market decisions corresponding to a different set of equilibrium operating conditions λ and \bar{f} . In the summary of results below the original market decisions are referred to as Set A and the alternative decisions as Set B.

<u>Market Decisions - Set A</u>	<u>Market Decisions - Set B</u>
$k_1 = 1.14$	$k_1 = 1.68$
$c_{12} = .68$	$c_{12} = .60$
<u>Equilibrium Operating Conditions</u>	<u>Equilibrium Operating Conditions</u>
$\lambda = 1.36$	$\lambda = 1.21$
$\bar{f} = 1.46$	$\bar{f} = 1.38$

Application of the cost model yielded the profit expressions P_A and P_B for the two sets of market decisions and equilibrium operating conditions. The resulting expression for the incremental profit $P_B - P_A$ is given below:

$$\Delta P = (P_B - P_A) = .553 + .158k_2 + .079k_7t_R + .316k_8$$

Conclusions. Market Decision Set B involves a higher selling price and smaller promotional expenditures than Set A. The result is a lower demand rate and smaller mean time in system. In particular, the average utilization of the system falls from .9 for Market Decision Set A to .8 for Market Decision Set B. Since the expression for ΔP is positive for any non-negative cost and inventory parameters (k_2, k_7, k_8 and t_R), it follows that Decision Set B is better than Decision Set A - independent of the system cost and inventory parameters. The reasons for this result are evident when the expression for ΔP is analyzed term by term. The first term includes the savings in promotional costs and increased income from sales resulting from the higher selling price. The second, third, and fourth terms reflect savings in raw materials, raw material inventory costs, and in-process inventory costs resulting from the lower sales volume with Decision Set B.

Step B5 - Variation of Reject Percentage.

The best set of market decisions from Step B⁴ was used to evaluate system performance for two different values of the reject percentage π .

In the summary of results below the system with $\pi=.05$ is referred to as System A, the system with $\pi=0$ as System B.

Market Decisions

$$k_1 = 1.68$$

$$c_{12} = .60$$

Equilibrium Operating Conditions <u>System A with $\pi=.05$</u>	Equilibrium Operating Conditions <u>System B with $\pi=0$</u>
$\lambda = 1.21$	$\lambda = 1.23$
$\bar{f} = 1.38$	$\bar{f} = 1.26$

The incremental profit obtained from the cost model is given by:

$$\Delta P = (P_B - P_A) = .033 + .044k_2 + .022k_7 t_R + .120k_8 - k_{12}(\pi=0) + k_{12}(\pi=.05)$$

Conclusions. The reduction in the reject percentage leads to smaller mean time in system and a slightly increased demand rate. The expression for ΔP is negative, i.e., System A with $\pi=.05$ is best, when:

$$k_{12}(\pi=0) - k_{12}(\pi=.05) > .033 + .044k_2 + .022k_7 t_R + .120k_8$$

Thus, in this case, the best alternative depends on the system cost and inventory parameters. Interpreting the expression above term by term, the condition under which System A is better than System B is when the additional costs of quality control exceed the increased income from sales resulting from the higher demand and the decreased cost of raw materials, raw materials inventory, and in-process inventory resulting from the elimination of rejects in System B.

Step B6 - Variation of System Capacity.

In evaluating two alternative system capacities, it was assumed that the system cost and inventory parameters were such that the system with $\pi=.05$ was found best in Step B5. In the summary of results below the system with one production station, one production worker, one inspection station, and one inspector is designated as System A. The market decisions and equilibrium operating conditions for System A are the results of the previous Steps. The system with two production stations, two production workers, two inspection stations, and two inspectors is designated as System B. The market decisions and equilibrium operating conditions for System B were obtained from the application of Steps B1 through B5 to the results obtained from simulation of the system with doubled capacity (Experiment 2).

Market Decisions - System A

$$c_1 = c_2 = n_1 = n_2 = 1$$

Market Decisions - System B

$$c_1 = c_2 = n_1 = n_2 = 2$$

$$k_1 = 1.68$$

$$k_1 = .14$$

$$c_{12} = .60$$

$$c_{12} = 1.21$$

Equilibrium Operating Conditions
for System A (from Experiment 1)

$$\lambda = 1.21$$

$$\bar{f} = 1.36$$

Equilibrium Operating Conditions
for System B (from Experiment 2)

$$\lambda = 2.42$$

$$\bar{f} = 1.35$$

The incremental profit obtained from the cost model is given by:

$$\Delta P = (P_R - P_A) = -2.305 - 1.273k_2 - k_3 - k_4 - k_5 - k_6 - .637k_7 t_R - 1.597k_8$$

Conclusions. System B with double capacity forces a much lower selling price and higher promotional expenditures to achieve the increased demand rate necessary to utilize the system. Since ΔP is negative for any non-negative cost and inventory parameters, it follows that System A is better than System B, i.e., doubling the system capacity while maintaining the average utilization level of .8 is undesirable - independent of the system cost and inventory parameters. The reasons for this result are evident when the expression for ΔP is analyzed term by term. The first term reflects increased promotional expenditures and decreased income from sales resulting from the lower selling price. The subsequent terms reflect increased costs of raw materials, equipment, labor, raw material inventories, and in-process inventories resulting from the higher sales volume in System B.

REFERENCES

- Baker, C.T., and Dzielinski, "Simulation of a Simplified Job Shop," Management Science, Vol. 6, No. 3, April, 1960.
- Bellman, Richard, Adaptive Control Processes A Guided Tour, Princeton University Press, Princeton, New Jersey, 1961.
- Buffa, Elwood S., Production-Inventory Systems: Planning and Control, Homewood, Illinois: Richard D. Irwin, Inc., 1968.
- Conway, R.W., "An Experimental Investigation of Priority Assignment in a Job Shop," Memorandum RM-3789-PR, The Rand Corporation, Santa Monica, California, February, 1964.
- Gere, W.S., Jr., "Heuristics in Job Shop Scheduling," Management Science, Vol. 13, No. 3, November, 1966.
- Jackson, James R., "Job Shop-Like Queueing Systems," Management Science, Vol. 10, No. 1, October, 1963.
- Johnson, S.M., "Optimal Two-and Three-Stage Production Schedules with Setup Time Included," Naval Research Logistics Quarterly, Vol. 1, No. 1, 1954.
- Nelson, Rosser T., "Labor and Machine Limited Production Systems," Management Science, Vol. 13, No. 9, May, 1967.
- Nelson, Rosser T., "A Simulation Study of Labor Efficiency and Centralized Labor Assignment Control in a Production System Model," Working Paper No. 126, Western Management Science Institute, University of California, Los Angeles, September, 1967.

Nelson, Rosser T., "Dual Resource Constrained Series Service Systems,"
Operations Research, Vol. 16, No. 2, March-April, 1968.

Palmer, D.S., "Sequencing Jobs Through a Multiprocess in the Minimum Total Time-A Quick Method of Obtaining A Near Optimum," Operational Research Quarterly, Vol. 16, No. 1, March, 1965.

Rowe, A.J., "Application of Computer Simulation to Sequential Decision Rules in Production Scheduling." Proceedings: Eleventh Annual Industrial Engineering Institute, University of California, Berkeley-Los Angeles, February, 1959.

Smith, Richard D., and Dudek, Richard A., "A General Algorithm for Solution of the n-Job, M-Machine Sequencing Problem of the Flow Shop,"
Operations Research, Vol. 15, No. 1, January-February, 1967.

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13 ABSTRACT This paper describes a research methodology for studying problems of analysis, design, and control in complex service systems. The proposed methodology is based upon the total systems point of view in the sense that the physical and decision-making aspects of the service system are considered and are related to environmental factors. The methodology was developed with two research areas in mind: (1) abstract studies of models of complex service systems and (2) field studies of complex service systems. An example is included to illustrate the first type of application. Field studies may be expected to present far more of a challenge because of the identification, modeling, and data requirements which will arise in actual situations. The methodological plan presented here focuses attention on these requirements as an integral part of systems analysis.		